Math 53, Discussions 116 and 118 **2022-8-29**

Parametric curves and their tangents II

Conceptual questions

Question 1. We've seen that

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

provided that the denominator is nonzero. How do we compute d^2y/dx^2 for a parametric curve?

Question 2. Let P be a point on a curve where the tangent is neither horizontal nor vertical. True or false:

- If $dy/dx > 0$ at P, then $dx/dy > 0$ also.
- If $d^2y/dx^2 > 0$ at P, then $d^2x/dy^2 > 0$ also.

Question 3. Given any single-variable function $f(x)$, you can view its graph as a curve in the xy-plane, with Cartesian equation $y = f(x)$. How can you parametrize this curve? Suppose that (x_0, y_0) is a point on the graph. If you compute the slope at this point using the §10.2 formula, do you get the same answer as you expect from Math 1A?

Computations

Problem 1. Find the equation of the tangent line to the parametric curve

$$x = 4e^{t-3} + 1 \qquad y = \sin(\pi t) + t \qquad -\infty < t < \infty$$

at the point $(5, 3)$.

Problem 2. You're probably sick of parametrizing circles at this point, but here's one parametrization of $x^2 + y^2 = 1$ that might be new to you, called *stereographic projection*.

The line passing through the points $(0, 1)$ and $(t, 0)$ intersects the unit circle at one point other than $(0, 1)$. Find the coordinates of this point, in terms of t.

In the parameter interval $-\infty < t < \infty$, does this parametrization trace out the entire circle?